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The Fuzzy Knowledge Representation in a First Logic

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ABSTRACT

Knowledge representation and processing within an expert system are rather conflicting characteristics, due to the fact that the increase in knowledge power representation reduces the system efficiency and validity. The use of variables in a knowledge representation formalism allows knowledge factorization. The first order predicates languages facilitate rigorously expressing complex knowledge, imposing appropriate reasoning techniques. The goal of this paper is to provide an extended framework for fuzzy knowledge representation and processing, used in a prototype system CFK.

1. Introduction

The CFK system consists of a compiled fuzzy rules base and the inference engine. The types of knowledge accepted by CFK system are [9,10]: i) variables (symbols always preceded by '?', as ?x, ?y, which will occur only in rules); ii) atomic constants (numbers or strings); iii) possibility distributions or fuzzy constants (symbols always preceded by the character '*' and used to represent imprecision); iv) logical operators. Possibility distribution can take any form. This complexity can cause a number of difficulties for the application of possibility theory. In economic practice, when the variables are numeric, it appears that a trapezoidal possibility distribution on continuous referentials is well suited. It can be represented through four parameters (g, d, φ, δ). Trapezoidal form of possibility distributions is preserved in most of the inference and calculation operations. All the fuzzy constants used in knowledge representation and modelling, for the synthesis of fuzzy reasoning algorithms, are represented by trapezoidal possibility distributions, such as $g \leq d$, $\varphi, \delta \geq 0$, called T-numbers. In most expert systems, the rules within the rules base are independent. Independence indicates clearly that no other rule interferes with the consequences inferred by any of the rules. Such rules are called normal rules. For fuzzy knowledge processing, the rules' independence hypothesis is not always valid. It is the case of incomplete rules which infer conclusions based on a collection of dependent rules. Managing uncertainty and imprecision is an important feature of an expert system, incorporating imprecise knowledge, and whose results are essentially addressed to the human decision maker [1,3,7]. There are different types of imprecision in a fuzzy expert system [12,17]: i) The confidence in a given rule and the relationship between a rule's premise and conclusion, called confidence in the inferred conclusion. Confidence in a rule can be seen as a form of fuzzy probability in the sense that the rule is usually true but not always. The numerical value of the confidence in a rule usually replaces linguistic values in most cases. ii) Rule priority allows taking into account its importance within the problem-solving process; iii) Confidence in the input data; iv) Imprecision in describing knowledge and data embedded into the management model; v) Filtering, unification and calculation of the inferred conclusion for the fuzzy case. The types of imprecision fall into three categories, according to where they occur: i) Confidence in a rule, confidence in the conclusion inferred and the priority of the rule are associated with the knowledge base; ii) Imprecision in describing knowledge and data can be found in the information provided by the expert or by the user, through various means of communication; iii) Filtering, unification and calculation of the inferred conclusion are specific to the inference engine. Since incorporating different types of imprecise knowledge reduces the response time of the expert system, is necessary the pre-processing of the fixed part of the management model. We exemplified on a reduced knowledge model the behaviour of our prototype CFK, respecting the formalisms and the analysed performances, as a real-time planning system [4,5,8,11].

Section 2 presents the CFK parameters used in all fuzzy processings, the section 3 refers to the GMP inference rule. The section 4 points out the general inference mechanism and at the end we propose some important conclusions.

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2.The CFK parameters used in all fuzzy processings

Fuzzy constants can occur both in facts and rules, and are always associated to fuzzy sets (T-numbers) through *constfaz* function. Within CFK we can equate the fuzzy set to a fuzzy constant. Undefined fuzzy constants are not allowed. A fuzzy constant has always a value corresponding to a continuous, trapezoidal and normalized fuzzy set. Using possibility distributions provides a unified framework for representing imprecision and uncertainty. Parameter ζ is used to measure fuzzy sets uncertainty ($0 \leq \zeta \leq 1$). If a fuzzy set is uncertain, the parameter ζ must be defined in *constfaz* function through a list (uncertain ζ). We admit that a completely uncertain fuzzy set ($\zeta = 1$) has no effect on system's behaviour. In contrast to facts, a reason is a structured list in which variables may occur. This signifies the presence of variables, atomic constants and of fuzzy constants within reasons' structure. In addition, the *reasons* (named also *motives*) may occur in both the conditional part and in rules' conclusion. Uncertainty is allowed in the conditional part and in the consequent of GMP inference rule, only if a particular linguistic model requires it. To process the fuzzy knowledge in CFK system, represented in a first order logic,we used the Generalized Modus Ponens (GMP) inference rule. This is justified because one of the great advantages of fuzzy expert systems is based on the fact that there is no need for a perfect fit between the rule's antecedent and a fact. Confluence between the knowledge pieces is achieved by using T-norms and ϕ-operators. Probabilistic reasoning's limits led to the development of uncertainty measures g, with the following properties: **i)** $g(F) = 0$, $g(T) = 1$; **ii)** if q is a logical consequence of p, then $g(q) \geq g(p)$; iii) $g(p \vee q) \geq \max(g(p), g(q))$ and $g(p \wedge q) \leq \min(g(p), g(q))$. If choosing the case of equality in these last two inequalities, we obtain the possibility Π and necessity N measures, where [2]:

$$\Pi(p \vee q) = \max(\Pi(p)), \Pi(q) \text{ and } N(p \wedge q) = \min(N(p), N(q))$$

Possibility measure represents the intersection degree of two fuzzy sets, denoted hereinafter by M and F (reason and fact). It is an optimistic measure and is defined as follows:

$$\Pi(M, F) = \sup_{u \in U} \min(\mu_M(u), \mu_F(u))$$

Necessity measure is defined based on possibility measure as follows:

$$N(M, F) = 1 - \Pi(\neg M, F) = 1 - \sup_{u \in U} \min\left(1-\mu_M(u), \mu_F(u)\right) = \inf_{u \in U} \max\left(\mu_M(u), 1-\mu_F(u)\right)$$

For $\Pi < 1$, $N = 0$ always takes place relation: $\Pi(M, F) \geq N(M, F)$, which indicate that N is a compatibility measure more demanding than Π. As a first conclusion, we may note that $\mathbf{N}(M, F) > 0 \Leftrightarrow$ *Supp*(M) $\supset$ *Ker*(F).

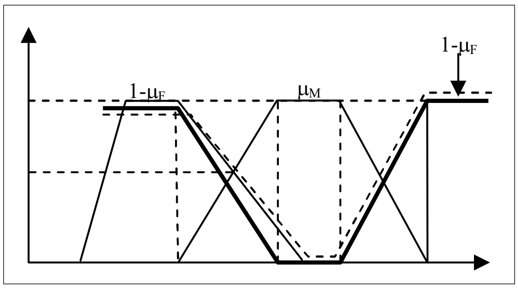

Figure 1. *The relationship between Π and N*

Figure 1 shows the significance of necessity measure, for the case:

Supp($\mu_M(u)$) $\cap$ *Supp*($\mu_F(u)$) $\neq \emptyset$, $N > 0$; *Ker*($\mu_M(u)$) $\cap$ *Ker*($\mu_F(u)$) $\neq \emptyset$.

Other situations that highlight the values of N compared with the relative position of μ_M and μ_F, respectively: *Supp*($\mu_M(u)$) $\cap$ *Supp*($\mu_F(u)$) $\neq \emptyset \Rightarrow 0<N<1$; *Ker*($\mu_M(u)$) $\subset$ *Ker*($\mu_F(u)$) $\neq \emptyset$; *Supp*($\mu_M(u)$) $\supset$ *Ker*($\mu_F(u)$); *Supp*($\mu_M(u)$) $\supset$ *Ker*($\mu_F(u)$); *Supp*($\mu_M(u)$)$\not\subset$ *Ker*($\mu_F(u)$) $\Rightarrow$ N<1 (very close to 1); *Supp*($\mu_F(u)$) $\supset$ *Supp* ($\mu_M(u)$); *Ker*($\mu_F(u)$) $\subset$ *Ker*($\mu_M(u)$); $N = 1 \Leftrightarrow$ *Supp*($\mu_F(u)$) $\subseteq$ *Ker*($\mu_M(u)$). For the CFK prototype developed and tested for a case study, the calculation of possibility and necessity measures is done in the "options" menu, parameters calculation.

3. GMP inference rule

A rule can be represented based on a *conditional possibility distribution*. Let a relationship between two variables X and Y respectively defined on the universes of discourse U and V, which is a restriction of possible values of Y when is assumed that the variable X takes certain values. It can be expressed through a function $\pi_{Y|X}$:V×U → [0,1]. This function is called conditional possibility distribution and is the degree of possibility that Y takes the value v, given that X takes the value u. If π_X is the possibility distribution that limits a priori the values of X, we can calculate the possibility distribution $\pi_{X|Y}$ which limits the values of the pair (u, v), using the two values of $\pi_{Y|X}$ and π_X. For (∀) v∈V, (∀) u∈U, is valid the relationship $\pi_{X|Y}(u, v) = \min(\pi_X(u), \pi_{Y|X}(v, u))$. By projection, is obtained the possibility distribution of $\pi_Y(v)$, which narrows the possible values of variable Y, given by:

$$(\forall)v\in V, \pi_Y(v)=\sup_{u\in U}\pi_{X_{|Y}}(u, v)=\sup_{u\in U}\min\left(\pi_X(u),\pi_{Y_{|X}}(v, u)\right)$$

In practice, one rule is not sufficient to fully describe a relationship, this requiring a collection of rules. If the relationship is represented by a single rule, then we get:

$$(\forall)v\in V,\mu_B(v)\geq\sup_{u\in U}\min\left(\pi_A(u),\pi_{Y_{|X}}(v, u)\right)$$

Inequality comes from the fact that is wanted "Y is B" when "Y is B", as soon as B′ is enclosed in B. There are several possible solutions, but we are essentially interested in the solution which is the largest (with respect to fuzzy sets inclusions), since this is the least restrictive (denoted $\pi'_{Y|X}(v, u)$), where:

$$\pi'_{X|Y}(v, u)=\begin{cases}0, \text{ if } \mu_A(u)\leq\mu_B(v)\\ \mu_B(v), \text{ else}\end{cases}$$

GMPinference rule *features* are: **i)** the conclusion B′ obtained by applying it, cannot be more specific or more restrictive than the rule consequent B, i.e. B ⊆ B′ if A′ is normalized; **ii)** the more restrictive is A′, the more restrictive is the conclusion B′, i.e. for A″⊆ A′, it follows B″ ⊆ B′; **iii)** if there is no perfect filtering between the given fact A′ and the rule antecedent A, then occurs a level of indeterminacy θ; **iv)** in practice, the rule is expressed as an operator which is represented at semantic level as a conditional possibility distribution. Introduced by Zadeh, the rule has the following solution:

$$(\forall)v\in V,\ \mu_{B'}(v)=\sup_{u\in U}\min\left(\pi_{A'}(u),\pi'_{Y_{|X}}\right)$$

In this case, this relationship can be re-written:

$$(\forall)\ v\in V,\ \mu_{B'}(v)=\sup_{u\in U}\min\left(\pi_{A'}(u),\pi_A(u)\overset{G}{\rightarrow}\pi_B(u)\right)\ \text{or simply } B'=A\ o\ (A\overset{G}{\rightarrow}B)$$

Let the calculation relationship for $\mu_{B'}(u)$ be a function that transforms μ_B into $\mu_{B'}$, during which occur two phenomena: **i)** a global level of indeterminacy of B′, for the values which are outside the support of B; **ii)** calculating the core of the inferred conclusion B′, which is determined by enlarging the core of the set B.

Definition 1. The degree of indetermination in the conclusion inferred through GMPrule, is the degree of possibility that A′ is not included in the support of set A, and the complement to 1 of this degree is to what extent it is necessary that Y takes values in A. The degree of indeterminacy is noted with θ and is calculated as follows:

$$\theta=\sup\{\mu_{A'}(u)\},\ u\in\{u\in U,\mu_A(u)=0\}$$

Let μ_A = tp(g, d, φ, δ) and $\mu_{A'}$= tp (g′, d′, φ′, δ′). We get:

$$\theta=\begin{cases}\max(\mu_{A'}(g-\varphi),(\mu_{A'}(d+\delta)),\ \text{if } Ker(A')\cap Supp(A)\neq\emptyset\\ 1,\ \text{else}\end{cases}$$

The degree of indetermination θ depends on the position of A′ with respect to A. $\theta_1=\mu_{A'}(g-\varphi)$, $\theta_2=\mu_{A'}(d+\delta)$ where *Supp*(μ_A) = (g-φ; d+δ) ⇒ θ = max{θ_1, θ_2} = θ_1. We may see that if g′≤g-φ or d′≥d+δ, then θ = 1, i.e. B′ is completely uncertain. The degree of indetermination occurs whenever *Supp*(A′) ⊈ *Supp*(A). If *Supp*(A) ⊇ *Supp*(A′), then θ = 0. We may see the degree of indeterminacy exactly as the maximum value in *Supp*(A′) which is not included in *Supp*(A).

Definition 2. The core of the inferred conclusion B′, denoted K, is to determine for which interval in V, $\mu_{B'}(v)$ = 1. According to relationship

$$\mu_{B'}(v)=1\Rightarrow\sup_{u\in U}\min(\pi_{A'}(u),\pi_A(u)\overset{G}{\rightarrow}\pi_B(v))=1\Rightarrow(\exists)\ u\in U,\ \text{so that } (\pi_{A'}(u)=1)\ \text{and } \pi_A(u)\leq\pi_B(v).$$

This is equivalent to the relation: K = inf ($\mu_A(u)$ | u ∈ {u ∈ U | $\mu_{A'}(u)$ = 1}}

We can see that $Ker(\mu_{A'}(u)) = [g', d']$, and obtain $K_1 = \mu_A(g')$ and $K_2 = \mu_A(d')$, which implies $K = \min(K_1, K_2)$. It follows that $\mu_B(v) \geq K$. Parameter K varies as the parameter θ, in connection with the position of A′ against the position of A. If $Ker(\mu_{A'}(u)) \subset Ker(\mu_A(u))$, then K = 1.

For $g' \leq g-\varphi$ or $d' \geq d+\delta$, it follows that K = 0. It is noted that the parameter K is zero iff $\theta = 1$, and K > 0 iff θ <1. Knowing the value of K, we can determine the core Ker(B′) of $\mu_{B'}$, which is the K-cut of B. For $\mu_B = (g_B, d_B, \varphi_B, \delta_B)$ and $Ker(\mu_{B'}) = [g_{B'}, d_{B'}]$, the core and the support of the inferred conclusion $\mu_{B'}$ are calculated as follows:

$$g_{B'} = f_{B'}^1(g_B, \varphi_B, K),\ d_{B'} = f_{B'}^2(d_B, \delta_B, K),\ \varphi_{B'} = f_{B'}^3\left(g_{B'}, g_B, \varphi_B, \theta\right),\ \delta_{B'} = f_{B'}^4(\delta_B, d_B, d_{B'}, \theta)$$

Functions $f_{B'}^1$, $f_{B'}^2$, $f_{B'}^3$ and $f_{B'}^4$ are similar to those presented in (Mazilescu 2012). The core of B′ is completely determined by the value of the parameter K.

For K = 1, we get Ker(B′) = Ker(B). Conversely, if 0<K<1, i.e. the core of A′ is not included in the core of A, but is included in A's support, then Ker (B′) $\supset$ Ker(B), which means that the result inferred by a rule is less accurate than the rule's conclusion. This is due to the filtering between condition A and the fact A′, which is not precise. The conclusion obtained by GMPalways leads to Ker(B′) $\supset$ Ker(B), i.e. the result inferred cannot be more precise than the rule's conclusion. The inferred conclusion $\mu_{B'}$ is determined finally with the relationship:

$$\mu_{B'} = \max((g_{B'}, d_{B'}, \varphi_{B'}, \delta_{B'}), \theta)$$

which shows that the basic element in the GMPcalculation is to evaluate parameters θ and K. If the rule's fact and condition are low compatible, the inference engine of CFK based on GMPrule induces an increase of the inferred conclusion's imprecision. These issues will arise in the case study for the fuzzy case of the flexible production system, when activating the meta-rules along with their chaining, demonstrating the effective use of all these theoretical results.

4. The compatibility of possibility distributions and fuzzy filtering

Possibility measure Π and necessity measure N, estimate the degree of compatibility between fact and reason, and the parameters K and θ serve for conclusion deduction.

- **Possibility distributions compatibility.** Assume that ((*f, *m $\rightarrow$ *c) *c′) where *m is the reason of rule *m $\rightarrow$ *c and *f is the fact, each expressed by (*constfaz* *m(tp g_m, d_m, φ_m, d_m)) and (*constfaz* *f(tp g_f, d_f, φ_f, d_f)). In order to obtain the conclusion *c′, should be known if the fact is compatible with the reason of the rule. If so, the GMP rule can be applied to infer the conclusion *c′. If the fact is uncertain ($\zeta \neq 0$), the fact $*f^{\zeta}$ is represented as: (*constfaz* $*f^{\zeta}$(tp g_f, d_f, φ_f, δ_f)(uncertain ζ)), μ_{*f_ζ} = **max** ($\mu_{*f}(u)$, ζ). For the latter case, the possibility and necessity measures Π and N are:

$$\Pi(*m, *f^{\zeta}) = \max(\Pi(*m, *f), \zeta) = \max\left(\sup_u \min(\mu_{*m}(u), \mu_{*f}(u)), \zeta\right)$$

$$N(*m, *f^{\zeta}) = \min(N(*m, *f), 1-\zeta) = \min\left(\inf_u \max(\mu_{*m}(u), 1-\mu_{*f}(u)), 1-\zeta\right)$$

Starting from the definition, the practical calculation of Π is:

$$\Pi(*m, *f) = \begin{cases} 1, \text{ if } Ker(*m) \cap Ker(*f) \neq \emptyset, \text{ where } Ker(*m) = [g_m, d_m],\ Ker(*f) = [g_f, d_f] \\ f\{(g_m - d_f)/(\varphi_m + \delta_f)\}, \text{ if } d_m - g_f > d_f - g_m \\ f\{(g_f - d_m)/(\varphi_f + \delta_m)\}, \text{ if } d_m - g_f < d_f - g_m \end{cases}$$

Necessity measure N(*m, *f) is the degree to which the fuzzy set *f is included into the fuzzy set *m. Generally, calculation of N is more complicated than the one of Π. A simple calculation method is based on separating the complement of the set *m. It fallows $N(*m, *f) = 1 - \Pi(\urcorner *m, *f) = 1 - \Pi(\max(E_s, E_d), *f)$, or:

$$N(*m, *f) = 1 - \max(\Pi(E_s, *f), \Pi(E_d, *f))$$

Thus defined and calculated Π and N respectively, we distinguish several classes of decreasing compatibility: i) Π(*m, *f) = 1 and N(*m, *f) = 1 equivalent to *full compatibility;* ii) Π(*m, *f) = 1 and N(*m, *f) > 0 equivalent with the values of *f are *compatible to some degree* with those of *m, but this is not fully certain; iii) $\Pi(*m, *f) \leq 1$ and N(*m, *f) = 0 (the values of *f are poorly compatible with *m, certainly); iv) Π(*m, *f) = 0 and N(*m, *f) = 0 *(total incompatibility).*

Even though Π and N measures estimate well the degree of compatibility between the fuzzy constants, these measures cannot be used directly to infer the conclusion for an inference engine based on the GMPrule. If the measures Π and N satisfy certain imposed thresholds, we only know that the fact *f managed to filter the reason *m. For the GMPrule calculating are also needed the parameters K and θ, whose significance is:

$$\theta(*m, *f) = \max(\mu_{*f}(g_m - \varphi_m), \mu_{*f}(d_m + \delta_m)),\ K(*m, *f) = \min(\mu_{*m}(g_f), \mu_{*m}(d_f))$$

In general, the reason C and the fact F contain more fuzzy constants, in the form:

$$Cont\_f(C) = (*p_1 ,...,*p_m) \text{ and } Cont\_f(F) = (*g_1,..., *g_m).$$

Possibility measure Π and necessity measure N, at the level of the entire reason and fact, are defined by: $\Pi(C, F) = \min \Pi(*p_i, *g_i)$, $N(C, F) = \min N(*p_i, *g_i)$, $(1 \leq i \leq m)$. Similarly we calculate the parameters θ and K at the level of the entire reason and fact:

$$\theta(C, F) = \max \theta(*p_i, *g_i),\ K(C, F) = \min K(*p_i, *g_i),\ 1 \leq i \leq m$$

The possibility distributions pattern-matching

Like for the classical case, the fuzzy pattern-matching aims to determine the sets of instances of the reasons. It is stronger than the classical one, due to its ability to process fuzzy knowledge, namely evaluating *the filtering degree* between a fuzzy reason and a fuzzy fact. One of the major differences between classical and fuzzy filtering is that classical pattern-matching determines whether a fact unifies or not with a reason, returning a binary result, while fuzzy f pattern-matching provides a result that is no longer binary. The conclusion is that the fact *filters more or less the reason.*

Let C a fuzzy reason and F a fuzzy fact. Reason C is represented as a nested list containing three types of data. The reason can be naturally represented through a tree, the sub-tree representing sub-lists, and the leaves are the knowledge pieces within the reason. Fuzzy fact F is also a nested list and contains only atomic and fuzzy constants (not variable). In order to filter a fact with a reason, is built a recursive algorithm, comparing the two associated trees to determine whether: **i)** the tree associated with the reason and the one associated with the fact have the same structure; **ii)** the leaves of the tree associated with the reason filter those of the fact tree.

Imprecise knowledge pattern-matching is the fundamental operation in terms of time (complexity). Achieving this objective involves the three cases: **a)** If the leaf of the tree associated with the reason (fam) is a variable (?x), it can filter any term, be it atomic constant (ca) or fuzzy constant (cf), represented by a corresponding leaf in the fact tree. This variable is then instantiated by that term. It is possible for the variable to take as value a fuzzy constant. Appears, in this case, the problem of *substitutions compatibility;* **b)** If the leaf of the reason tree is an atomic constant, then there are two possible filtering situations: either the atomic constant corresponds exactly with the one from the fact tree or the filtering fails; **c)** If the leaf of the reason tree is a fuzzy constant defined with *constfaz* function, there are two possibilities: either the fuzzy constant filters a number or another fuzzy constant from the fact tree, or must be assessed whether the reason's fuzzy constant is compatible with the data (the fact), which can be a fuzzy number or constant. We synthesize the presented concepts in the form of a filtering algorithm, presented below.

Algorithm1 *(Fuzzy reason-fact* pattern-matching*)*

If **(fam** =?x) *then* **{fam** can filter any term}
{Term = atomic constant or fuzzy constant, ?x instantiates itself with a term}
{arises the problem of substitutions compatibility}
else if **(fam** = **ca)**
then if **(fam** = **faf)** *then* filtering fails
else filtering fails
End
else **{fam** = **cf}**
{either **cf** filters a fuzzy number or constant from the fact tree, or must be assessed to what extent the reason's fuzzy constant is compatible with the fact, which can be a **ca** or another **cf**}
End

The most interesting problem is the one in which fam = cf and faf = cf, raising the issue of the pattern-matching degree between two fuzzy sets. Are preferred two scaling measures in order to estimate the compatibility between M and F. These measures are Π(M, F) and N(M, F). The Π(M, F) estimates to what extent it is probable for M and F to refer to the same value u, in other words Π(M, F) is the degree of intersection of the fuzzy set corresponding to the possible values of F. The N(M, F) estimates to what degree is necessary (i.e. certain) that the value referred by F to be among the ones compatible with Π, or N(M, F) is the degree of inclusion of the possible values from F into the set of values compatible with M. Always take place the relations: $\Pi(M, F) \geq N(M, F)$, $N(F, F) \geq 1/2$, $N(Supp (F), F) = 1$.

We can also check the following properties: **i)** $\Pi(M, F) = 0 \Leftrightarrow Supp(M) \cap Supp(F) = \emptyset$; **ii)** $\Pi(M, F) = 1 \Leftrightarrow Ker(M) \cap Ker(F) \neq \emptyset$; **iii)** $N(M, F) = 1 \Leftrightarrow Supp(F) \subseteq Ker(M)$.

The algorithm based on the iteration of condition/fact filtering is inefficient because of the numerous redundancies. Assume T, a limited time for the filtering between a condition C_j and a fact F, the cost for determining $IC(C_j)$ is $T\times| BF |$. The cost in time for the elementary filtering of the rules base is lower than the value: T×K×|BF|×|BR|, where K is the maximum number of conditions per rule. The complexity of condition-fact filtering is O(|BR|×|BF|).

Knowing the significance of the four parameters Π, N, θ, K, we may consider the problem of choosing the reasonable thresholds for the possibility and necessity measures, to determine the facts that do not filter at all the reasons. This choice must be made in close accordance with the values of θ and K parameters, knowing the links between these pairs of parameters. For the GMPrule, we consider that the associated T-numbers are: (*Constfaz* *m(tp g_m, d_m, φ_m, δ_m)), (*constfaz* *c(tp g_c, d_c, φ_c, δ_c)) and (*constfaz* *f(tp φ_f, d_f, φ_f, δ_f)); this rule proves the following properties:

Proposition 1 a) $K = 0 \Leftrightarrow \theta = 1$; **b)** $K > 0 \Leftrightarrow \theta < 1$; **c)** Conclusion *c′ inferred through GMPis totally uncertain ($\mu_{*c'} = 1$) $\Leftrightarrow \theta = 1$.
Choosing the right thresholds for measures Π and N is based on the next results.
Proposition 2 N(*m, *f) > 0 $\Leftrightarrow \theta < 1$. This derives from the observation that: $\theta < 1 \Leftrightarrow$ *Supp*(*m) ⊃ *Ker*(*f) ⇔ N(*m, *f) >.
We may notice the fulfilment of the condition which ensures the inequality N(*m, (f)> 0 (i.e. θ <1, equivalent with *Supp*(*m) ⊃ *Ker*(*f)) (Figure 2).

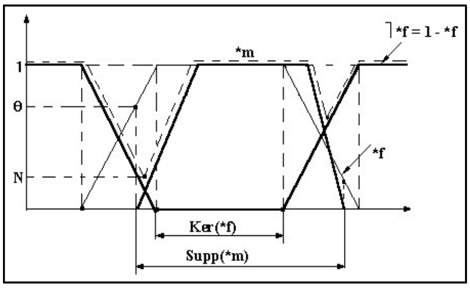

Figure 2. Graphical relationship between N and θ

For the necessity measure and the GMPinference rule, are proved the following equivalent relations: **a)** $N > 0 \Leftrightarrow \theta < 1 \Leftrightarrow K > 0$; **b)** N(*m, *f) = 0 ⇔ θ = 1; **c)** If N = 1, then θ = 0. Inverse property ($\theta = 0 \Rightarrow N = 1$) is not always true. Outlining a demonstration for the direct, we get: N = 1 ⇔ *Ker*(*m) ⊒ *Supp*(*f) ⇒ *Supp* (*m) ⊒ *Supp* (*f) ⇔ θ = 0.

The above results show that the necessity measure and the degree of indeterminacy θ are closely linked. Also can be revealed a quantitative relation between θ and N. According to a previous observation, the necessity measure is defined by: N(*m, *f) = 1 – max Π(E_s, *f), Π(E_d, *f)), showing clearly that the terms Π(E_s, *f) and Π(E_d, *f) determine the value of necessity measure. If N(*m, *f) is known, the following two situations may be deducted: 1) Π(E_s, *f) = 1-N and Π(E_d, *f) ≤ 1-N or 2) Π(E_s, *f) ≤ 1-N and Π(E_d, *f) = 1-N.
First, we consider case 1), where Π(E_s, *f) ≥ Π(E_d, *f), understanding the fact that the value of possibility measure between the fuzzy set E_s and the fact (fuzzy) *f determines the necessity measure because:

$$N(*m, *f) = 1 - \mathbf{max}\ (\Pi(E_s, *f), \Pi(E_d, *f)) = 1\ \mathbf{max}(1-N, \Pi(E_d, *f) \leq 1-N)) = 1-(1-N) = N$$

It suffices to consider the left side, E_s, of the complement of ⌉*m. We have the situation when Π(E_s, *f) ≥ Π(E_d, *f), and the significance of Π(E_s, *f) is defined by: Π(E_s, *f) = sup min $\mu_{Es}(u_0)$, $\mu_{*f}(u_0)$).

Following the above observations, we get:
a) $0 < \theta_d < \theta_s < 1$, Π($E_s$, *f) ≥ Π($E_d$, *f), $\theta = \max(\theta_s, \theta_d) = \theta_s$
b) $0 < \theta_d < \theta_s < 1$, Π($E_s$, *f) ≤ Π($E_d$, *f), $\theta = \max(\theta_s, \theta_d) = \theta_d$

For N(*m, *f) = 1 - max (Π(E_s, *f), Π(E_d, *f)), with max (E_s, E_d) = ⌉*m, we get:
a) if Π(E_s, *f) ≥ Π(E_d, *f), then $\theta = \theta_s$, b) Π(E_s, *f) ≤ Π(E_d, *f), then $\theta = \theta_d$

We may point out that the values of parameters θ and K can be shown as a type of filtering between a fact and the rule's antecedent, when applying GMP rule. Necessity N and the degree of indetermination play a similar role in evaluating the basic filtering degree. Thus, it becomes natural to select as threshold for the necessity measure in the filtering stage (for the inference engine that uses GMP) the value N > 0, as it ensures consistency between fuzzy filtering and GMP inference rule. Following the above analysis, and correlating it with the values of possibility and necessity measures also used in the fuzzy variables binding stage, we settle as a criterion for fuzzy filtering within CFK, the simultaneous fulfilment of conditions: *Π(*m, *f) = 1 and N(*m, *f) > 0*, which ensure the preservation of normality for the possibility distributions derived from the logical inferences.

Fuzzy unification in CFK system

Fuzzy unification aims at checking the consistency of fuzzy substitutions, in which variables can be associated with fuzzy sets. Fuzzy unification is one of the basic problems in fuzzy expert systems, and can be represented as follows: Let R be a rule with Cond(R) = $\{C_1, ..., C_k\}$. After fuzzy condition-fact filtering, if each condition C_i filters a fact F_i (in the fuzzy sense), then there is a fuzzy substitution σ_i so that $F_i \cong \sigma_i \cdot C_i$, and eventually the four parameters Π_i, N_i, θ_i and K_i (if there are fuzzy constants in the rule's reason and in its associated fact). Let ?v a variable of the rule, which we assume that appears n times in the conditional part of the rule (counted only the occurrences within the reasons and not within the predicates). We use $?v_i$ to note the occurrence i of the variable ?v. In this case, all the occurrences of ?v in the global condition of the rule can be represented by the list $\{?v_1, ?v_2, ..., ?v_n\}$. After the fuzzy condition-fact filtering, each variable $?v_i$ will certainly be associated with a term t_i, which can be an atomic constant or a fuzzy constant, and which can be denoted schematically by: $\{t_1/?v_1, t_2/?v_2, ..., t_n/?v_n\}$. All the different variables of a rule are independent. Each variable may appear several times in a rule, each occurrence of the variable being independent of other occurrences. Almost all expert systems preserve this hypothesis.

Fuzzy unification consists of two stages: i) the *consistency checking* for the items in list $\{t_1/?v_1, t_2/?v_2, ..., t_n/?v_n\}$ with respect to a given criteria. Checking substitutions consistency resides in a symbolic comparison operation. For the crisp case, this operation is relatively easy. For the case of fuzzy facts, the variables will be associated with fuzzy terms, and the consistency checking is more complicated; ii) the *fuzzy substitutions composition:* starting from the set $\{t_1, ..., t_n\}$, using a given criteria and following the calculation algorithm, is built a new value t_λ, which will be associated with the variable as a result of the composition. To eliminate any confusion, we use $?v_\lambda$ to denote the variable ?v after the fuzzy unification, which is represented by: $\{t_1/?v_1, t_2/?v_2, ..., t_n/?v_n\} \rightarrow \{t_\lambda/?v_\lambda\}$, where t_λ shall be calculated.

5. Conclusions

Formal logic reasoning is embedded in rule-based production systems. Experts'operational knowledge is expressed as rules, each rule gathering information relative to its launching conditions and information on the resulting effects. The inference engine performs its reasoning dynamically, by chaining the basic cycles, which involves two phases: *evaluation* and *execution*. In the evaluation phase the engine determines if, within the rules base, there are rules to be activated, using the current state, and also the effective set of these rules. The inferential chains determine the *reasoning process.* In the execution phase the engine launches the rules found in the evaluation stage. The complexity of the domain of the problem that an expert system must solve, involve different reasoning strategies [6,13,14,15,16,18]: i) if the state space is small and the knowledge involved is low, then the search can be exhaustive and the reasoning can be monotonous (use only one type of reasoning); ii) if the data or knowledge is complex, is justified to combine indexes from multiple sources, and may be used probabilistic models or fuzzy models; iii) for the time-varying knowledge is necessary to include certain temporal attributes and appropriate reasoning techniques; iv) for large state spaces is possible to use generation, hierarchical testing, as well as abstraction; v) if the sub-problems or the agents interact, can be used restriction propagation techniques; vi) if necessary to use incomplete models, can be used the plausible reasoning and the backtracking; vii) when a reasoning model is too weak, are used multiple reasoning lines; viii) if the knowledge base is ineffective (size, complexity, operating time, etc.), is useful to apply the knowledge compilation technique, used within CFK expert system, presented in this paper. We may outline the following conclusions: i) the formalism chosen for knowledge representation is strong enough to support the representation of some types of knowledge underlying the management decisions synthesis. It has the advantage of factorizing the knowledge, which substantially reduced the size of fuzzy rules base. Also, this knowledge representation method is more appropriate to express some types of knowledge similar to those commonly used in the decision synthesis through natural language; ii) the management fuzzy model for the problem presented was developed incrementally, as was embedded in the model sufficient domain knowledge, resulting from the limitations observed for the crisp case. Is it possible to constantly adapt the management fuzzy model through simulations, often rather difficult; iii) the inferential subsystem based on fuzzy logic, designed and presented throughout this paper, solves the management situations correctly, both from the computational point of view and in terms of the semantics of the conclusions inferred through the chosen inference rule; iv) modeling the economic problems and the expert system as systems with logical events allowed the qualitative analysis of management expert system.

References

1. *Bergmann, G., Rath, I., Varro, D. (2009). Parallelization of Graph Transformation Based on Incremental Pattern Matching, Proceedings of the Eighth International Workshop on Graph Transformation and Visual Modeling Techniques (GT-VMT 2009), Electronic Communications of the EASST, Volume 18, ISSN 1863-2122.*
2. *Dubois, D., Prade, H. (1987). Théorie des possibilités, applications à la représentation des connaissances en informatique, Masson, 2-ième edition.*
3. *Eleftherakis, G. and Cowling, A.J. (2003). An agile formal development methodology, in Tigka K. and Kefalas P. (Eds.), Proceedings of the First South-East European Workshop on Formal Methods, Thessaloniki, Greece, 20 November 2003, Thessaloniki, pp. 119-137.*

4. Fagiolo, G., Dosi, G., and Gabriele R. (2004). Matching, bargaining, and wage setting in an evolutionary model of labor market and output dynamics, Advances in Complex Systems, vol. 7, pp. 157-186.
5. Forgy, C.L. (1982). Rete: A Fast Algorithm for the Many Pattern/Many Object Pattern Match Problem, Artificial Intelligence (19), pp. 17-37.
6. Galbraith, J. K. (1968). Le nouvel Etat Industriel: essai sur le systeme économique américain, Editions Galimard, Paris.
7. Galland S. & al. (2005). L'implication des experts dans un processus de prise de decision, Actes du Colloque ATELIS - Atelier d'Intelligence Stratégique. Poitiers, pp 61-71.
8. Jacobi, I., Radul A. (2010). A RESTful Messaging System for Asynchronous Distributed Processing, http://dig.csail.mit.edu/2010/Papers/WS-REST/wsrest2010.pdf (accessed 25th December, 2012).
9. Mazilescu, V. (2010). Characterizing an Extended Fuzzy Logic System with Temporal Attributes for Real-Time Expert Systems, The 10th WSEAS International Conference on Systems Theory and Scientific Computation (ISTASC'10), Taipei, TAIWAN, p.159-165.
10. Mazilescu, V. (2011). An Intelligent System for a Resource Allocation Problem based on Fuzzy Reasoning, Computer Technology and Application, David Publishing Company, From Knowledge to Wisdom, ISSN 1934-7332, p. 247-255.
11. Mazilescu, V. (2012). A Knowldege Management System Embedded in the New Semantic Technologies, Chapter 1, pp. 1-22, in New Research on Knowledge Management Technologies, edited by Huei-Tse Hou, INTECH, Janeza Trdine 9, 51000, Rijeka, Croatia, ISBN 978-953-51-0074-4.
12. Qian, D. (1992). Representation and Use of Imprecise Temporal Knowledge in Dynamic Systems, Fuzzy Sets and Systems, Vol. 50, no 1, Aug. 25, p. 59-77.
13. Reaz, A., Boutaba. R., (2007). Distributed Pattern Matching for P2P Systems, IEEE Journal on Selected Areas in Communications, http://bcr2.uwaterloo.ca/~rboutaba/Papers/Journals/JSAC-07.pdf (accesed 2th December, 2012).
14. Shvartzshnaider, Y., Ott, M., Levy. D. (2010). Publish/Subscribe on Top of DHT using Rete Algorithm. Lecture Notes in Computer Science, Vol. 6369/2010.
15. Wagner, T., Phelps, J., Qian, Y, Albert, E. and Beane, G. (2001). A Modified Architecture for Constructing Real-Time Information Gathering Agents, Agent Oriented Information Systems (AOIS), 2001.
16. Walzer, K., Breddin, T., and Groch, M. (2008). Relative temporal constraints in the Rete algorithm for complex event detection. In DEBS '08: Proceedings of the second international conference on Distributed event-based systems, pages 147-155, New York, NY, USA, 2008. ACM.
17. Walzer, K., Groth, M., Breddin, T. (2008). Time to Rescue - Supporting Temporal Reasoning in the Rete Algorithm for Complex Event Processing, Lecture Notes in Computer Science, Volume 5181/2008, pp. 635-642.
18. Westerhoff, F.H. (2004). Multi-assets Market Dynamics, Macroeconomics Dynamics vol. 8, pp. 596-616.