Comparison of the EWMA and GARCH Models with Respect to Estimation of the Exchange Rates Volatilities

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ABSTRACT

In recent years, the financial system has been evolving and developing at a rapid pace. Both the investors and the other market players aspire to know whether there is volatility in the market and to determine the structure of such fluctuations in case they exist. In addition to this, the accurate volatility estimation models are required to be able to conduct better risk management, portfolio management and option pricing in financial markets. In this context, the field of research in volatility estimation has been developing quickly. Ultimately, whichever is used, the fundamental purpose of volatility prediction models is to accurately estimate volatility. In this study, MA, EWMA, GARCH (1,1) and IGARCH models have been used to conduct volatility predictions with respect to GBP/TRY and EUR/TRY exchange rates between 04.01.2007 and 31.12.2009. ME and RMSE tests have been used to evaluate the reliability levels of the volatility estimates. According to the test results, it has been determined that EWMA model has yielded better estimates than GARCH(1,1) and IGARCH models in terms of estimating the volatilities of exchange rates.

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1. Introduction

The global financial markets have entered into a phase of high risk after dissolution of the Bretton Woods system in 15th of August, 1971. Enormous public deficits, inflation and economic events like the oil embargo by Organization of the Petroleum Exporting Countries-OPEC have fueled the volatility in the financial system. Besides, the transition to a floating rate system has introduced the concept of exchange risk to the countries. The procuring cause of exchange rate risk is the excessive level of volatility among exchange rates. Especially, one of the most crucial steps of Value at Risk calculations employed by financial institutions for the evaluation of market risks is determining the coefficient of volatility to be used. Accordingly, the concept of volatility and the volatility estimation models that are frequently used in the financial markets will firstly be explained in this study. In the last section, MA (moving average), EWMA (Exponentially Weighted Moving Average), GARCH (1,1) (Generalized Autoregressive Conditional Heteroskedasticity) and IGARCH (Integrated GARCH) models have been used to conduct volatility predictions with respect to GBP/TRY and EUR/TRY exchange rates between 04.01.2007 and 31.12.2009. ME (Mean Error) and RMSE (Root Mean Square Error) reliably tests have been employed to compare the obtained results. Microsoft Excel and FIA (Financial Instrument Analyzer) have been utilized for the calculations and formation of the graphs (Financial Instrument Analyzer is a financial decision supporting system developed by RiskActive, a financial consulting firm, which can perform calculations pertaining to various fixed or variable income financial instruments and derivatives by utilizing internationally accepted financial engineering models and techniques).

2. The Concept of Volatility

Poon (2005: XV) statistically describes volatility as the level of standard error. On the other hand, Ploeg (2006) defines volatility as the uncertainty of the fluctuations in assets prices. Volatility level in the financial markets varies greatly in time. The standard deviation of the financial factor or variance analysis can be used as methodologies to measure this variation (Jorion, 2005: 371). However, a complete consensus has not been maintained between the experts with respect to the issue of which volatility estimation model should be chosen since different conclusions might be obtained by different volatility models. In this day and age, many financial institutions are in need of calculating the VAR that their portfolios are exposed to or testing their capital adequacy ratios (Suganuma, 2000:1). Volatility estimations are intensively used in asset management, portfolio management or pricing of derivatives instruments and thereby play a major role in the financial markets (McMillan and Speight, 2004: 1). The historical values for the GBP/TRY and EUR/TRY exchange rates taken from CBRT (Central Bank of the Republic of Turkey) for the analyzed period of 3 years

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spanning 2007-2009 has been demonstrated in Graph 1 drawn in Excel (GBP/TRY: Turkish Liras per 1 British Pound and EUR/TRY: Turkish Liras per 1 Euro).

Graph 1. Graph of Historical GBP/TRY and EURO/TRY Exchange Rates

When the evolutionary trend for the GBP/TRY and EUR/TRY exchange rates from 2007 to 2009 are examined in Graph 1, it is seen that these two rates have converged towards each other despite possessing significantly different values with respect to Turkish Lira at the beginning of 2007. In the fourth quarter of 2009, an increasing trend in EUR and a decreasing trend in GBP have been observed.

In Graph 2, the variation of the returns obtained by taking the logarithmic differences of the exchange rates is a better indicator of the volatility between the exchange rates. These graphics demonstrate the extent of variation in returns attributable to the rises and falls in exchange rates. The level of fluctuations arising in times of financial distress is especially noteworthy.

3. An Overview of the Literature

Numerous studies about volatility models could be found by investigating the related literature. For instance, Tse (1991: 290) has utilized the EWMA, GARCH and ARCH models on the data from the Japanese securities exchange and has pointed out EWMA as the best available method. In their study of the volatility of the ISE (Istanbul Stock Exchange) 30 Index, Korkmaz and Aydin (2002: 2) have recorded that GARCH model had yielded more accurate results compared to EWMA. According to the volatility estimation performed by Kumar (2006: 2) about the Indian stock exchange, it has been found that the EWMA model provides more reliable results compared to the volatility estimates by GARCH and the same pattern of success is also seen in the GARCH model oriented at the foreign exchange (Forex) markets. In the study by Minkah (2007:III) examining the volatilities of S&P 500 and various indices from other countries, it has been stated that GARCH model leads to better predictions in comparison to EWMA and historical volatility especially with respect to shorter term projections.

As could be seen in other prior studies as well, the primary inconvenience concerns the question of which model provides the better results. Hence, the objective of this study is to determine the reliabilities of the volatility estimations by the GARCH (1,1) and EWMA models, which are both used by many financial institutions and recommended for usage by RiskMetrics as well. In addition, IGARCH model has also been included in the calculations to provide a better explanation for the EWMA model, because the lambda coefficient used in this model needs to be optimized in time and IGARCH model is used in this optimization. In this way, the comparisons will be statistically more meaningful. However, it should not be forgotten that the predictions generated by both of these models are quite close to each other.
4. Volatility Models

The volatility models frequently preferred and utilized in the financial markets can be listed as the historical, implied, EWMA (Exponentially Weighted Moving Average) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models (Ray, 2003: 1). In the historical volatility approach, a retrospective analysis of the past data is carried out; however, the variance and covariance estimations based on historical volatility may often lead to inaccurate results (Giannopoulos, 2000). The reason is that actual volatility of future results may diverge from the volatility estimations derived from the historical data since \( \sigma \) is assumed to be constant throughout the time series. In this study, the calculations were conducted based on the EWMA and GARCH models.

4.1. MA (Moving Average)

An estimation of average historical volatility with respect to a time frame of \( n \) periods is the square root of the standard error terms:

\[
\sigma_{\text{MA}}^2 = \frac{1}{n} \sum_{s=t-n+1}^{t-1} (r_s - \mu)^2
\]

In the model, \( r_s \) denotes the asset returns in time period \( s \) and \( \mu \) denotes the average asset return. The advantage of this estimation methodology is its easy application and that it excludes any factor beyond the period of the associated volatility. In the variance estimation as per the MA model, every observation assigned an equal weight. Thus, the volatility will either go through a sharp increase or severely drop in case of a financial shock.

4.2. EWMA (Exponentially Weighted Moving Average)

EWMA has been developed on the principle that asset returns are symmetrical and independently distributed and incorporates the assumption of time dependent volatility (Bolgün and Akçay, 2005). In order to conduct its calculations, the model utilizes two fundamental parameters, time and lambda, respectively. \( \lambda \) is the coefficient that specifies the sensitivity of the predicted volatility to the past data (Candan and Özün, 2006:101). The value for this coefficient takes on a value between 0 and 1. For example, in the RiskMetrics application (1996: 97) developed by JP Morgan, it is recommended that a coefficient of 0.94 should be used for daily data and 0.97 for monthly data. In the model, the weights assigned to each realized data point exponentially decreases going further into the past (Candan and Özün, 2006:101). Consequently, the higher impact of the latest variations is clearly observed on the estimations. In this context, the EWMA model could be given by the following equation:

\[
\sigma_{\text{EWMA}}^2 = \lambda \sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2
\]

In the model, \( \sigma_{n}^2 \) denotes the volatility at the \( n \)th day (period), \( u_{n-1}^2 \) stands for the return on the day right before the \( n \)th day, \( \sigma_{n-1}^2 \) is the volatility for the day prior to the \( n \)th day and \( \lambda \) is a constant coefficient between 0 and 1.

4.3. GARCH (Generalized Autoregressive Conditional Heteroskedasticity)

GARCH approach is a generalized version of the ARCH methodology that had been developed by Engle (1982) and it is widely known as the GARCH model proposed by Bollerslev (1986). The equation describing the GARCH(1,1) model is shown below (Hull, 2000: 373).

\[
\sigma_{\text{GARCH}}^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2
\]

In the equation above, \( \sigma_{n}^2 \) denotes the volatility at the \( n \)th day (period); \( \omega = yV \), where \( V \) is the long term mean volatility, \( u_{n-1}^2 \) stands for the return on the day right before the \( n \)th day, \( \sigma_{n-1}^2 \) is the volatility for the day prior to the \( n \)th day. The major difference between the GARCH and EWMA models is that the former uses \( \sigma_{n-1}^2 \) and \( u_{n-1} \) values as well as the long term mean variance \( V \) (Candan and Özün, 2006:102). In the GARCH model, the inequality \( \alpha + \beta \pi \) must be satisfied. Otherwise, the weight assigned to the long term mean variance will become practically negative (Hull, 2000: 373). The values for approaching unity is an indication of the volatility being persistent for a longer term and more permanent, whereas values diverging away from unity indicates the volatility to be shorter term and temporary (Candan and Özün, 2006: 103).

A sound estimation of volatility plays a key role with respect to financial decision making in our day for the financial institutions and other market participants. In practice, methodologies like MA, EWMA, ARCH, GARCH and stochastic volatility models are adopted. Volatility estimation models like GARCH and EWMA demonstrate a stronger performance compared to those methods which assume that the variance stays constant in the face of all the changes and developments in the financial markets. Certain characteristics
possessed by financial asset returns preclude the accurate predictions of volatility. For instance, (Tsay, 2005) has proposed that the following distributional properties have contributed to the success of volatility estimation models such as GARCH which take into account the fact that the variance changes in time:

- Highly flattened distributions (leptokurtic)
- Wide and frequent volatility clusters
- The volatility values being confined to a particular interval
- Factors such as leverage possessed by the associated instruments (extremely high values for both falls and rises)

Essentially, this situation has laid the foundation towards the development of new volatility models. For instance, EGARCH model has been devised in order to detect the asymmetry imposed on the volatility due to highly negative or positive shocks in asset returns (Tsay, 2005: 99).

Thereby, for the purpose of more realistic and consistent volatility predictions, using the Exponentially Weighted Moving Average - EWMA and GARCH methodologies is strongly recommended (Hull, 2000).

4.4. IGARCH (Integrated Generalized Autoregressive Conditional Heteroskedasticity)

IGARCH is a limited version of the GARCH method. The sum of the continuous parameters in the model should be unity:

\[
\sum_{i}^{\beta} + \sum_{j}^{\alpha} = 1
\]  

(4)

This process is defined IGARCH as long as \( \alpha + \beta = 1 \) holds and the IGARCH process assumes that the variation of the returns demonstrate a persistency effect over the variance even though they are generally discontinuous (Franses, 1995:113). Similar to ARIMA models, the primary characteristic of the IGARCH model is the influences of the squared magnitudes of the past shocks, as implied by the equation \( \eta_{i-j} = \alpha_i^2 - \sigma_{f-i}^2 \), where \( i > 0 \) and \( \alpha_i^2 \) demonstrates the persistency of volatility (Tsay, 2005:122). The fact that the GARCH process has a unit root indicates that the volatility will eventually revert back to its mean. Furthermore, it is assumed that the conditional variance will display persistency over the shocks. Lastly, the GARCH model is employed for Lambda optimization in the EWMA model (Bolgün and Akçay, 2009:372) and the coefficient of \( \beta \) in the IGARCH model is also known as the decay factor in the EWMA model, which takes on a value between 0 and 1. The IGARCH model is shown below (Tsay, 2005:122):

\[
a_i = \sigma_i^2, \quad \sigma_i^2 = \alpha_0 + \beta_0 \sigma_{i-1}^2 + (1 - \beta_1) \alpha_{i-1}^2
\]

where \( \alpha_0 \) is a constant and \( \sigma_i^2 \) is an estimate for the coefficient of previous error terms in the model.

5. Data

In this study, the daily exchange rates for GBP/TRY and EUR/TRY for the period from 04.01.2007 to 31.12.2009 have been used. Before proceeding with the volatility calculations, the statistical results derived from the series are given in Table 1.

<table>
<thead>
<tr>
<th>EUR/TRY</th>
<th>GBP/TRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0002</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.0840</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.1355</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0001</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.3300</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.6665</td>
</tr>
</tbody>
</table>

The skewness and kurtosis values given in Table 1 explain whether the sample space is normally distributed or not. Accordingly, the distribution is considered to be normal if the skewness value is 0 and the kurtosis value is 3. As these measures take on values further away from 0 and 3, respectively, the distribution will become either skewed or flattened/peaked and thus depart from normality. This situation is especially critical for the risk measurement models which make an a priori assumption of normal distribution, since the reliability of the risk measurement results is reduced if the distribution is not normal.
Estimations of Volatility For the Period of 2007-2009

In Table 2, the estimations for mean, maximum and minimum annual volatilities for the returns on GBP/TRY and EUR/TRY exchange rates during the years of 2007, 2008 and 2009 have been displayed.

Table 2. A condensed outlook of the annualized volatility estimations for the daily returns of the GBP (£) and Euro (€) exchange rates given by the EWMA, GARCH (1,1) and IGARCH models

<table>
<thead>
<tr>
<th>Years</th>
<th>GBP (£)</th>
<th></th>
<th></th>
<th></th>
<th>EUR (€)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GARCH (1,1) (annual)</td>
<td>EWMA (annual)</td>
<td>IGARCH (annual)</td>
<td>GARCH (1,1) (annual)</td>
<td>EWMA (annual)</td>
<td>IGARCH (annual)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007 (max)</td>
<td>0.3302 (15.08.2007)</td>
<td>0.2387 (15.08.2007)</td>
<td>0.2863 (16.08.2007)</td>
<td>0.3216 (05.10.2007)</td>
<td>0.2415 (05.10.2007)</td>
<td>0.3841 (08.10.2007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007 (min)</td>
<td>0.0853 (17.01.2007)</td>
<td>0.0812 (05.07.2007)</td>
<td>0.0827 (12.06.2007)</td>
<td>0.0763 (21.02.2007)</td>
<td>0.0680 (23.02.2007)</td>
<td>0.0716 (22.02.2007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007 (mean)</td>
<td>0.1142</td>
<td>0.1177</td>
<td>0.1299</td>
<td>0.0853</td>
<td>0.1032</td>
<td>0.1315</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008 (max)</td>
<td>0.4803 (27.10.2008)</td>
<td>0.3743 (27.10.2008)</td>
<td>0.4314 (28.10.2008)</td>
<td>0.4563 (28.10.2008)</td>
<td>0.3674 (30.10.2008)</td>
<td>0.5359 (30.10.2008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008 (min)</td>
<td>0.1042 (26.02.2008)</td>
<td>0.1146 (26.02.2008)</td>
<td>0.1073 (27.02.2008)</td>
<td>0.0736 (11.01.2008)</td>
<td>0.0890 (18.01.2008)</td>
<td>0.0668 (07.01.2008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008 (mean)</td>
<td>0.1586</td>
<td>0.1928</td>
<td>0.1837</td>
<td>0.1240</td>
<td>0.1739</td>
<td>0.1806</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009 (max)</td>
<td>0.3399 (06.01.2009)</td>
<td>0.3064 (06.01.2009)</td>
<td>0.3124 (07.01.2009)</td>
<td>0.2764 (08.01.2009)</td>
<td>0.2726 (09.01.2009)</td>
<td>0.3153 (09.01.2009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009 (min)</td>
<td>0.0819 (30.12.2009)</td>
<td>0.0760 (31.12.2009)</td>
<td>0.0751 (03.09.2009)</td>
<td>0.0719 (29.12.2009)</td>
<td>0.0637 (29.12.2009)</td>
<td>0.0650 (30.12.2009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009 (mean)</td>
<td>0.1341</td>
<td>0.1609</td>
<td>0.1544</td>
<td>0.1190</td>
<td>0.1032</td>
<td>0.1215</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The volatility forecasts for the historical GBP/TRY and EUR/TRY exchange rates derived from the MA, EWMA, GARCH (1,1) and IGARCH models are displayed in Graph 3 and Graph 4, respectively.

Graph 3. Volatility estimations for the returns on GBP (£) exchange rates as per the MA, EWMA, GARCH (1,1) and IGARCH models
An Assessment of the Volatility Estimations

The performance of the volatility predictions depends on the conformity between the actual data and the estimations by the model. In other words, if \( X_t \) denotes the model estimate, \( X_t \) denotes the actual result and the estimation error will be given by \( \varepsilon_t = X_t - X_t \). Specifically, \( X_t \) and \( X_t \) stand for the estimated and realized conditional variances in this study. \( X_t \) should be therefore either \( \sigma_t \) or \( \sigma_t^2 \). In practice, the sum of the squares of the daily returns will yield the conditional variance. In all models based on a particular time series, the volatility predictions depend on past data. Consequently, the predictions never forecast any upcoming development or shocks.

The volatility predictions concerning the financial markets are built on numerous preformed factors, hence the value derived from the volatility model after a shock impacts the system will depend on the accurate formation of the restructured factors specified for the following days. In this context, higher weights are assigned to the error terms in case of a recent shock in the formulas determining conditional volatility or variance (\( \sigma_t^2 \)). When the error estimates are measured from the variance, the confidence interval for the mean error values will become much wider, which complicates the detection of significant deviations between the volatility estimation models (Poon, 2005).

The error statistics determined for the forecasted values by the volatility models is used for the purpose of comparing the deviations between the forecasted values and the realized outcomes, after disregarding the signs and also compensating for the magnitudes of the error terms. Essentially, the contemporary valuation methodologies based on error statistics are Mean Error - ME, Mean Square Error – MSE and Root Mean Square Error – RMSE tests. In this study, the results derived by the Mean Error and Root Mean Square Error methods have been favored among these available techniques, since RMSE method has been preferred by both Minkah (2007) and Figlewski (2004) in the reliability tests for their volatility forecasts. The RMSE test could be given by the equation below:

\[
\sqrt{\frac{1}{N} \sum_{t=1}^{N} \varepsilon_t^2} = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (\hat{\sigma}_t - \sigma_t)^2},
\]

In this notation \( \sigma \) denotes the actual volatility at time \( t \) and \( \hat{\sigma} \) denotes the volatility forecast for time \( t \).

The notation for the Mean Error (ME) test is given below:

\[
\frac{1}{N} \sum_{t=1}^{N} |\varepsilon_t| = \frac{1}{N} \sum_{t=1}^{N} |\hat{\sigma}_t - \sigma_t|.
\]

When comparing different volatility models according to their reliability tests, the coefficient of MA volatility has been used as the base model or the realized volatility.
historical calculations, forecast based calculations and various volatility estimation models such as EWMA and GARCH are used to forecast volatilities. Still, the primary objective is accurate estimation of volatility regardless of the model being used.

Within the scope of this study, volatilities for GBP/TRY and EUR/TRY exchange rates have been estimated by the assistance of MA, EWMA GARCH and IGARCH models. In our day, the GARCH model is one of those volatility estimation models that manage to explain the volatility values most realistically. It is one of the widely accepted and preferred volatility estimation models especially with respect to the financial markets where the returns fluctuate greatly and excessive clustering is present. Besides, the EWMA and GARCH models are both frequently employed in RiskMetrics calculations. The significance of this study will be further underscored if we contemplate that even the tiniest differences in forecasts may become highly critical in the current financial markets.

It has been recorded that the models have attained reliable forecasts with respect to their estimations. Moreover, the models have generated close results with each other and have displayed similar distributions with respect to their time intervals. Comparing their volatility estimates, the EWMA model has demonstrated better performance according to the RMSE reliability test for both of the exchange rates compared to the GARCH (1,1) and IGARCH models. On the other hand, the GARCH (1,1) model has put forth a better performance regarding the daily EUR/TRY exchange rates as per the ME reliability test. However, it should be noted that RMSE test is more frequently employed to measure reliability level in volatility estimations.

In conclusion, the current financial markets are directly influenced by a highly diverse group of global risk factors, primarily led by political and economical developments. Concordantly, the EWMA and GARCH models will become one of the strongest determining factors with respect to eliminating or predicting the exposure to such influences as well as conducting more accurate risk measurements.

6. A General Assessment and Conclusions

As the global financial system is rapidly evolving, determining the volatilities of the financial instruments in the financial markets carries great significance. The financial markets tend to become more technical, technology intensive and depend more on econometric calculations each day. In this context, historical calculations, forecast based calculations and various volatility estimation models such as EWMA and GARCH are used to forecast volatilities. Still, the primary objective is accurate estimation of volatility regardless of the model being used.

In evaluation of the error performances, smaller values for RMSE are preferred. Accordingly, the EWMA model has demonstrated better performance with respect to volatility forecasts for both of the exchange rates compared to the GARCH (1,1) and IGARCH models. Nonetheless, the GARCH (1,1) model has displayed a better predicting power according to ME performance test for the EUR/TRY exchange rates. On the other hand, a similar difference has not been observed for the GBP/TRY exchange rates.

### Table 3. The performance results for the volatility tests

<table>
<thead>
<tr>
<th></th>
<th>EUR/TRY</th>
<th>GBP/TRY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ME</td>
<td>RMSE</td>
</tr>
<tr>
<td>GARCH(1.1)</td>
<td>0.0549</td>
<td>0.0665</td>
</tr>
<tr>
<td>IGARCH(1.1)</td>
<td>0.0590</td>
<td>0.0738</td>
</tr>
<tr>
<td>EWMA</td>
<td>0.0558</td>
<td><strong>0.0652</strong></td>
</tr>
</tbody>
</table>

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