Statistical Analysis Through Factors Path Method

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Abstract
The present research reflects the importance in statistics of the „Factors Path Method" (FPM), which provides modern techniques for the reflection of the factorial influences over of the variation in time or space presented by the phenomenons from economy. The purpose of this paper is the research of the shape concerning the model described through the road achieved by factors, with other words if the shape is an arc of curve or if the shape is a liner trend. Also, this paper presents how it uses „Factors Path Method" for to calculate what influence it had in relative sizes, respectively absolute sizes, the variation of nominal G.D.P., respectively the change of the level concerning actual G.D.P., over of the dynamic for the level of G.D.P. Prices Deflator for U.S.A., in 2006 face of 1996

Key-Words: Gross Domestic Product (GDP); Factor Path Method (FPM), influences; dynamic

JEL Code: C1, C12, C2.

1. Introduction

I present through this paper the study of the viability concerning the true opinion through which it reflects the description of the separated road of *X* and *Y* factors which influence any phenomenon from economy. This study cans be considered a personal scientific contribution through the manner of comparison of the opinions manifested lengthways of the time over the model concerning the road of the influences factors of any economic phenomenon.

In Section 2 we cheked up and confirmed the hypothesis which suposes that the road described, by the factors of influences concerning the GDP Prices Deflator, follows an arc of curve. Also, in Section 2 of this paper we cheked up and confirmed another hypothesis which suposes that the multiplication, respectively the sum of the factorial influences manifested over the level of GDP Prices Deflator, in the situation in which the model concerning the road of the factor follow an exponential trend, there are equal with the multiplication, respectively the sum of the factorial influences achieved over the same phenomenon and calculated in the situation in which the road of the factors reflected a model of liner trend.

The state of the art in this domain is represented by two essential researchs. The first research belongs to Ioan Florea who reflects the opinion through which the model concerning the road of the factors is under the shape of the arc of curve [3]. The second research belongs to Şaifulin and Seremet and she certifies the opinions which present that the road of each factor, *X*, respectively *Y*, is under the shape of the model of liner trend [7]. The conclusions of the paper appear in Section 3.

2. The model concerning the road of the factors

For to check up the viability of the opinion elaborated by Ioan Florea, which expresses the existence of the model concerning the road of the factors under the shape of the arc of curve: $X=X(t)$, $Y=Y(t)$ [3], we will consider the evolutions of the nominal G.D.P., respectively actual G.D.P., from U.S.A. in the period 1996–2006, represented in the table number 1, where the actual

G.D.P. is calculated in the prices of 2000 year. Also, in the frame of the statistical dates reflected, there are presented the values of G.D.P. Prices Deflator for U.S.A. in each period of time. So, we formulated the next hypothesis of the research:

- H_0^{DF-arc}: the assumption of the existence as model of trend of the road followed by each factor of influence of the dynamic concernig G.D.P. Prices Deflator for U.S.A., respectively nominal G.D.P. and actual G.D.P., from at P_0 to P_t, as a arc of curve, according to the opinion of Ioan Florea;
- H_0^{DF**}: the assumption of the inexistence concernig the diference between the values of the levels which express the relative total variation, respectively absolute, of G.D.P. Prices Deflator for U.S.A. as following of the influences of the both factors: nominal G.D.P. and actual G.D.P., in 2006 face of 1996, and determined in the situation in which the model for the road of each factor reflects a arc of curve, and the respective values calculated in the conditions in which the roads of the factors reflect in a separated mode a model of liner trend.

For to identify which opinion it confirms, we must to establish the model of trend on which it presents the road on each factor, *X*, respectively *Y*, between P_0 and P_t. In this sense, we will apply the method of the coefficients of variation right modality of selection for the best model of trend.

Table 1. The evolutions of nominal G.D.P., respectively actual G.D.P. and of G.D.P. Deflator between 1996-2006

Years	Nominal G.D.P. (USD billions) (x_i)	Actual G.D.P. (in the prices of 2000 year) (USD billions) (y_i)	I_{dfl}^{p}	
				(%)
1996	7.816,9	8.328,9	0,938527297	93,85
1997	8.304,3	8.603,5	0,965223455	96,52
1998	8.747,0	9.066,9	0,964717819	96,47
1999	9.268,4	9.470,3	0,978680717	97,87
2000	9.817,0	9.817,0	1,000000000	100,00
2001	10.128,0	9.890,7	1,023992235	102,40
2002	10.469,6	10.048,8	1,041875647	104,19
2003	10.960,8	10.301,0	1,064052034	106,41
2004	11.712,5	10.703,5	1,094268230	109,43
2005	12.455,8	11.048,6	1,127364553	112,74
2006	13.246,6	11.415,3	1,160425044	116,04
Total	112.926,9			

Source: http://www. bea.gov/national/x/s/gdp/ev/x/s – U.S. Department of Commerce, Bureau of Economic Analysis, Gouvernement of U.S.A

We observe that in the table number 1, the values of nominal G.D.P and actual G.D.P. calculated in the prices of the year 2000, respectively of G.D.P. Prices Deflator for U.S.A. growed in the period 1996–2006. If we apply „Factors Path Method", we decompose the dynamic of the next indicator [4]:

$$I_{dfl}^{p} = \frac{G.D.P._{nominal}}{G.D.P._{actual}}$$

where: I_{dfl}^{p} = the index of deflation concerning G.D.P Prices;

$G.D.P._{nominal}$ = nominal Gross Domestic Product;

$G.D.P._{actuall}$ = actual Gross Domestic Product

If we note $G.D.P._{nominal} = x$ and $G.D.P._{actual} = y$, then G.D.P. Prices Deflator has the next formula:

$$I_{dfl}^{p} = \frac{x}{y} = \varphi(x, y)$$

Also, we note with *1* the curent year *2006* and with *0* the year of base *1996*.

So, *x(0)* = 7.816,9 USD billions; *y(0)* = 8.328,9 USD billions;

x(1) = 13.246,6 USD billions; *y(1)* = 11.415,3 USD billions.

In the case of the **geometrical decomposition** of the dynamic of U.S.A. G.D.P. Prices Deflator, (I_{dfl}^{p}), with the help of „Factors Path Method", we obtain the next factorial indexes [3]:

$$I_{1/0}^{\varphi(x/y)} = e^{\int_{(P_0P_1)} \frac{\varphi_x^{'}(x,y)}{\varphi(x,y)}dx} \quad \text{and} \quad I_{1/0}^{\varphi(y/x)} = e^{\int_{(P_0P_1)} \frac{\varphi_y^{'}(x,y)}{\varphi(x,y)}dy}$$

Yet,

$$\varphi_x^{'}(x,y) = \frac{1}{y} \quad \text{and} \quad \varphi_y^{'}(x,y) = -\frac{x}{y^2}$$

Then:

$$I_{1/0}^{\varphi(x/y)} = e^{\int_{(P_0P_1)} \frac{1}{x}dx} \quad ; \quad I_{1/0}^{\varphi(y/x)} = e^{-\int_{(P_0P_1)} \frac{1}{y}dy}$$

But, the calculation of the factorial indexes imposes to establish the model of trend which will show us the type of function after which the factors *X* and *Y* follow the road from to P_0 towards P_1. if we apply the method of the coeficients of variation, we will consider the year from the middle of the series of each factor, right origin of calculation, while through the achievement of the substitution $\sum_{i=-m}^{m} t_i = 0$, we will obtain the next parametres:

● *in the case of* X *factor:*

- if we formulate the null hypothesis $H_0^{'}$: which reflects the assumption of the existence of the model of trend concerning *X* factor as a liner function $x_{t_i} = a + b \cdot t_i$, then the parametres *a* and *b* of the adjusted function of the degree *I* will be expressed with the help of the system:

$$\begin{cases} n \cdot a = \sum_{i=-m}^{m} x_i \\ b \cdot \sum_{i=-m}^{m} t_i^2 = \sum_{i=-m}^{m} t_i \cdot x_i \end{cases}$$

Consequently,

$$a = \frac{\sum_{i=-m}^{m} x_i}{n}$$

and

$$b = \frac{\sum_{i=-m}^{m} t_i \cdot x_i}{\sum_{i=-m}^{m} t_i^2}$$

We will obtain for the parametres *a* and *b* the next values:

$$a = \frac{112.926,9 \cdot 10^9}{11} = 10.266,08182 \text{ billions \$}$$

$$b = \frac{56.688,4 \cdot 10^9}{110} = 515,34909 \text{ billions \$}$$

So, the coefficient of variation calculated in the situation of the adjusted function of the first degree will be:

$$v_I = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^I \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^I \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{1.771,1 \cdot 10^9}{112.926,9 \cdot 10^9} \cdot 100 = 1,57\%$$

- in the case of the alternative hypothesis $H_1^{"}$: which mentions the formulation of the assumption concerning the existence of the model of trend for X factor as a parabolical function of second degree, $x_{t_i} = a + b \cdot t_i + ct_i^2$, the parametres *a*, *b* and *c* of the adjusted function of the second degree can be calculated by means of the system:

$$\begin{cases} n \cdot a + c \sum_{i=-m}^{m} t_i^2 = \sum_{i=-m}^{m} x_i \\ b \cdot \sum_{i=-m}^{m} t_i^2 = \sum_{i=-m}^{m} t_i \cdot x_i \\ a \cdot \sum_{i=-m}^{m} t_i^2 + c \sum_{i=-m}^{m} t_i^4 = \sum_{i=-m}^{m} t_i^2 \cdot x_i \end{cases}$$

Consequently,

$$a = \frac{\sum_{i=-m}^{m} t_i^4 \cdot \sum_{i=-m}^{m} x_i - \sum_{i=-m}^{m} t_i^2 \cdot \sum_{i=-m}^{m} t_i^2 \cdot x_i}{n \cdot \sum_{i=-m}^{m} t_i^4 - (\sum_{i=-m}^{m} t_i^2)^2}; \; b = \frac{\sum_{i=-m}^{m} t_i \cdot x_i}{\sum_{i=-m}^{m} t_i^2} \quad c = \frac{n \cdot \sum_{i=-m}^{m} t_i^2 \cdot x_i - \sum_{i=-m}^{m} t_i^2 \cdot \sum_{i=-m}^{m} x_i}{n \cdot \sum_{i=-m}^{m} t_i^4 - (\sum_{i=-m}^{m} t_i^2)^2}$$

Thus, the parametres *a*, *b* and *c* will reflected the values:

$$a = \frac{1.958 \cdot 112.926,9 \cdot 10^9 - 110 \cdot 1.144.088 \cdot 10^9}{11 \cdot 1.958 - (110)^2} = 10.093,36622 \text{ billions \$}$$

$$b = \frac{56.688,4 \cdot 10^9}{110} = 515,34909 \text{ billions \$}$$

$$c = \frac{11 \cdot 1.144.088 \cdot 10^9 - 110 \cdot 112.926,9 \cdot 10^9}{11 \cdot 1.958 - (110)^2} = 17,27156 \text{ billions \$}$$

As effect, the coefficient of variation obtained in the case of the adjusted function of the second degree will be:

$$v_{II} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{1.213,2 \cdot 10^9}{112.926,9 \cdot 10^9} \cdot 100 = 1,07\%$$

- in the situation of the alternative hypothesis $H_2^{"'}$: through which we suppose the existence of the model of trend for X factor as an exponential function $x_{t_i} = ab^{t_i}$, then the parametres *a* and *b* of the adjusted exponential function can be determinated by means of the system:

$$\begin{cases} n \cdot \lg a = \sum_{i=-m}^{m} \lg x_i \\ \lg b \cdot \sum_{i=-m}^{m} t_i^2 = \sum_{i=-m}^{m} t_i \cdot \lg x_i \end{cases}$$

So,
$$\lg a = \frac{\sum_{i=-m}^{m} \lg x_i}{n}$$

and
$$\lg b = \frac{\sum_{i=-m}^{m} t_i \cdot \lg x_i}{\sum_{i=-m}^{m} t_i^2}$$

Then, we will obtaine:

$$\lg a = \frac{143{,}06460196}{11} = 13{,}0058729$$

$$\lg b = \frac{2{,}40362142}{110} = 0{,}021851103$$

Consequently, in the situation of the existence concerning the adjusted exponential function, the coefficient of variation will be:

$$v_{\exp} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{1.184{,}5 \cdot 10^9}{112.926{,}9 \cdot 10^9} \cdot 100 = 1{,}05\%$$

We observe that: $v_{\exp.} = 1{,}05\% < v_{II} = 1{,}07\% < v_I = 1{,}57\%$

As effect, the road described of *X* factor, from to P_0 towards P_1, is an exponential trend with the shape $x_{t_i} = ab^{t_i}$, with other words it confirms the alternative hypothesis $H_2^{'''}$.

- **in the case of *Y* factor:**

- if we formulate the null hypothesis $H_0^{'}$: through which it specifies that the model of trend concerning *Y* factor it's a liner function $y_{t_i} = a + b \cdot t_i$, then the parametres *a* and *b* of the adjusted function of the first degree can be calculated by means of the next formulas:

$$a = \frac{\sum_{i=-m}^{m} y_i}{n} \quad \text{and} \quad b = \frac{\sum_{i=-m}^{m} t_i \cdot y_i}{\sum_{i=-m}^{m} t_i^2}$$

Hence, the parametres *a* and *b* will present the next values:

$$a = \frac{108.694{,}5 \cdot 10^9}{11} = 9.881{,}318182 \text{ billions \$}$$

$$b = \frac{32.015{,}4 \cdot 10^9}{110} = 291{,}0490909 \text{ billions \$}$$

Thus, the coefficient of variation in the case of the adjusted function of the first degree will be:

$$v_I = \left[\frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^I \right|}{n} : \frac{\sum_{i=-m}^{m} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^I \right|}{\sum_{i=-m}^{m} y_i} \cdot 100 = \frac{1.095{,}5 \cdot 10^9}{108.694{,}5 \cdot 10^9} \cdot 100 = 1{,}008\%$$

- in the case of the alternative hypothesis $H_1^{''}$: which suposes the existence of the model of trend concerning Y factor as a parabolical function of the second degree, $y_{t_i} = a + b \cdot t_i + ct_i^2$, then the parametres a, b and c of the adjusted function of the second degree can be calculated by means of the formulas:

$$a = \frac{\sum_{i=-m}^{m} t_i^4 \cdot \sum_{i=-m}^{m} y_i - \sum_{i=-m}^{m} t_i^2 \cdot \sum_{i=-m}^{m} t_i^2 \cdot y_i}{n \cdot \sum_{i=-m}^{m} t_i^4 - (\sum_{i=-m}^{m} t_i^2)^2}; \quad b = \frac{\sum_{i=-m}^{m} t_i \cdot y_i}{\sum_{i=-m}^{m} t_i^2}; \quad c = \frac{n \cdot \sum_{i=-m}^{m} t_i^2 \cdot y_i - \sum_{i=-m}^{m} t_i^2 \cdot \sum_{i=-m}^{m} y_i}{n \cdot \sum_{i=-m}^{m} t_i^4 - (\sum_{i=-m}^{m} t_i^2)^2}$$

So, the parametres a, b şi c will reflected the values:

$$a = \frac{1.958 \cdot 108.694{,}5 \cdot 10^9 - 110 \cdot 1.084.923{,}2 \cdot 10^9}{11 \cdot 1.958 - (110)^2} = 9.904{,}882284 \text{ billions \$}$$

$$b = \frac{32.015{,}4 \cdot 10^9}{110} = 291{,}04909 \text{ billions \$}$$

$$c = \frac{11 \cdot 1.084.923{,}2 \cdot 10^9 - 110 \cdot 108.694{,}5 \cdot 10^9}{11 \cdot 1.958 - (110)^2} = -2{,}35641 \text{ billions \$}$$

Consequently, in the situation of the existence concerning the adjusted function of the second degree, the coefficient of variation will be:

$$v_{II} = \left[\frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{II} \right|}{n} : \frac{\sum_{i=-m}^{m} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{II} \right|}{\sum_{i=-m}^{m} y_i} \cdot 100 = \frac{1100{,}2 \cdot 10^9}{108.694{,}5 \cdot 10^9} \cdot 100 = 1{,}012\%$$

- if we formulate the alternative hypothesis $H_2^{'''}$: through which we supose that the model of trend concerning Y factor it's an exponential function $y_{t_i} = ab^{t_i}$, then the parametres a and b which belong to the adjusted exponential function can be obtained through the formulas:

$$\lg a = \frac{\sum_{i=-m}^{m} \lg y_i}{n} \quad \text{and} \quad \lg b = \frac{\sum_{i=-m}^{m} t_i \cdot \lg y_i}{\sum_{i=-m}^{m} t_i^2}$$

Hence, we will obtaine:

$$\lg a = \frac{142{,}9216051}{11} = 12{,}99287319$$

$$\lg b = \frac{1{,}41841444}{110} = 0{,}012894676$$

In this way, the coefficient of variation obtained in the case of the adjusted exponential function will be:

$$v_{\exp} = \left[\frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{\exp} \right|}{n} : \frac{\sum_{i=-m}^{m} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| y_i - y_{t_i}^{\exp} \right|}{\sum_{i=-m}^{m} y_i} \cdot 100 = \frac{1.078,7 \cdot 10^9}{108.687,2 \cdot 10^9} \cdot 100 = 0,99\%$$

We observe that: $v_{\exp.} = 0,99\% < v_I = 1,008\% < v_{II} = 1,012\%$

In conclusion, the road reflected by Y factor from to P_0 towards P_1 it's an exponential trend with the shape $y_{t_i} = a \cdot b^{t_i}$, which it has as effcet the confirmation of the hypoyhesis $H_2^{'''}$.

Consequently, it verifies the hypothesis H_0^{DF-arc} and it's true Ioan Florea opinion. This opinion specifies the existence as model of trend of the road followed by each factor, X, respectively Y, from to P_0 towards P_t, as a arc of curve, in this research we can say as it's a arc of exponential followed by each factor of influence of G.D.P. Prices Deflator for U.S.A., in the curent period 2006 face of the base period 1996.

If we apply „Factors Path Method" in the case of the *geometrical decomposition* concerning the dynamic of G.D.P. Prices Deflator for U.S.A., (I_{dfl}^p), we observe that, because X factor varies after an exponential function of shape $x_{t_i} = ab^{t_i}$, we will obtaine [3]:

$$x(0) = a \quad \text{şi} \quad x(1) = a \cdot b = x(0) \cdot b, \quad \text{namely} \quad b = i_{1/0}^x$$

Thus,

$$x = x(0) \cdot (i_{1/0}^x)^t$$

On the other syde,

$$lnx=lna+tlnb \Rightarrow \frac{1}{x} \cdot dx = \ln b \cdot dt$$

Accordingly,

$$dx = x(0) \cdot (i_{1/0}^x)^t \cdot \ln i_{1/0}^x \cdot dt$$

Because the model of trend for Y factor is an exponetial function $y_{ti} = a \cdot b^{t_i}$, we obtaine in analogous mode:

$$y = y(0) \cdot (i_{1/0}^y)^t$$

and

$$dy = y(0) \cdot (i_{1/0}^y)^t \cdot \ln i_{1/0}^y \cdot dt$$

In continuation, we will calculate the next integral:

$$\int_{(P_0P_1)} \frac{dx}{x} = \int_{x(0)}^{x(1)} \frac{dx}{x(0) \cdot (i_{1/0}^x)^t} = \int_0^1 \frac{x(0) \cdot (i_{1/0}^x)^t \cdot \ln i_{1/0}^x}{x(0) \cdot (i_{1/0}^x)^t} dt = \ln i_{1/0}^x \cdot \int_0^1 dt = \ln i_{1/0}^x \cdot t =$$

$$= \ln i_{1/0}^x = \ln \frac{x(1)}{x(0)} = \ln \frac{13.246,6 \cdot 10^9}{7.816,9 \cdot 10^9} = \ln 1,694610395 = 0,527452859$$

Therefore,

$$I_{1/0}^{\varphi(x/y)} = e^{\int_{(P_0P_1)} \frac{\varphi_x'(x,y)}{\varphi(x,y)} dx} = e^{\int_{(P_0P_1)} \frac{dx}{x}} = e^{0,527452859} = 1,694610395 \cong 1,6946 \text{ sau } 169,46\ \%$$

If we proceed in analogous mode, we will obtaine:

$$\int_{(P_0P_1)} \frac{dy}{y} = \ln i_{1/0}^y \cdot t = \Big|_0^1 \ln i_{t/0}^y = \ln \frac{y(1)}{y(0)} = \ln \frac{11.415,3 \cdot 10^9}{8.328,9 \cdot 10^9} = \ln 1,370565141 = 0,315223166$$

As effect,

$$I_{1/0}^{\varphi(y/x)} = e^{\int\limits_{(P_0P_1)} \frac{\varphi_y^{'}(x,y)}{\varphi(x,y)}dy} = e^{-\int\limits_{(P_0P_1)} \frac{dy}{y}} = e^{-0,315223166} = 0,729626028 \cong 0,7296 \ \text{sau} \ 72,96\ \%$$

We observe that the value of G.D.P. Prices Deflator for U.S.A. growed with 69,46 % under the influence of the change concernign the nominal G.D.P. in the year 2006 face of the year 1996, while as effect of the variation for actual G.D.P. in the same period of time, the value of G.D.P. Prices Deflator subtracted with - 27,04 %. If we apply „Factors Path Method" in the case of the *arithmetical decomposition* of the dynamic concerning G.D.P. Prices Deflator for U.S.A., (I_{dfl}^{p}), we will calculate the values in absolutes sizes of the separated influences of the factors: *X*, respectively *Y*, over G.D.P. Prices Deflator [3]:

$$\Delta_{1/0}^{\varphi(x/y)} = \int\limits_{(P_0P_1)} \varphi_x^{'}dx = \int\limits_{(P_0P_1)} \frac{1}{y}dx$$

and

$$\Delta_{1/0}^{\varphi(y/x)} = \int\limits_{(P_0P_1)} \varphi_y^{'}dx = -\int\limits_{(P_0P_1)} \frac{x}{y^2}dy$$

Consequently,

$$\Delta_{1/0}^{\varphi(x/y)} = \int\limits_{(P_0P_1)} \varphi_x^{'}dx = \int\limits_{(P_0P_1)} \frac{1}{y}dx = \int\limits_{x(0)}^{x(1)} \frac{1}{y(0)\cdot(i_{1/0}^{y})^t}dx = \frac{x(0)\cdot \ln i_{1/0}^{x}}{y(0)}\int\limits_{0}^{1}\left(\frac{i_{1/0}^{x}}{i_{1/0}^{y}}\right)^t dt = \frac{x(0)\cdot \ln i_{1/0}^{x}}{y(0)}\int\limits_{0}^{1} e^{t\ln\left(\frac{i_{1/0}^{x}}{i_{1/0}^{y}}\right)}dt =$$

$$= \frac{x(0)\cdot \ln i_{1/0}^{x}}{y(0)\cdot \ln\left(\frac{i_{1/0}^{x}}{i_{1/0}^{y}}\right)}\cdot[e^{\ln\left(\frac{i_{1/0}^{x}}{i_{1/0}^{y}}\right)}-1] = \frac{x(0)\cdot \ln i_{1/0}^{x}}{y(0)\cdot(\ln i_{1/0}^{x} - \ln i_{1/0}^{y})}\cdot(e^{\ln i_{1/0}^{x} - \ln i_{1/0}^{y}}-1) =$$

$$= \frac{7.816,9\cdot 10^9\cdot \ln 1,694610395}{8.328,9\cdot 10^9\cdot(\ln 1,694610395 - \ln 1,370565141)}\cdot(e^{\ln 1,694610395 - \ln 1,370565141}-1) = 0,5514808$$

Also, we will obtaine:

$$\Delta_{1/0}^{\varphi(y/x)} = \int\limits_{(P_0P_1)} \varphi_y^{'}dy = -\int\limits_{(P_0P_1)} \frac{x}{y^2}dy = -\int\limits_{y(0)}^{y(1)} \frac{x(0)\cdot(i_{1/0}^{x})^t}{y^2(0)\cdot(i_{1/0}^{y})^{2t}}dy = -\frac{x(0)}{y^2(0)}\cdot\int\limits_{0}^{1}\frac{(i_{1/0}^{x})^t}{(i_{1/0}^{y})^{2t}}\cdot y(0)\cdot(i_{1/0}^{y})^t\cdot \ln i_{1/0}^{y}dt =$$

$$= -\frac{x(0)\cdot \ln i_{1/0}^{y}}{y(0)}\int\limits_{0}^{1}\left(\frac{i_{1/0}^{x}}{i_{1/0}^{y}}\right)^t dt = -\frac{x(0)\cdot \ln i_{1/0}^{y}}{y(0)}\int\limits_{0}^{1} e^{t\cdot\ln\frac{i_{1/0}^{x}}{i_{1/0}^{y}}}dt = -\frac{x(0)\cdot \ln i_{1/0}^{y}}{y(0)\cdot \ln\left(\frac{i_{1/0}^{x}}{i_{1/0}^{y}}\right)}\cdot(e^{\ln\frac{i_{1/0}^{x}}{i_{1/0}^{y}}}-1) =$$

$$= -\frac{x(0)\ln i_{1/0}^{y}}{y(0)(\ln i_{1/0}^{x} - \ln i_{1/0}^{y})}(e^{\ln i_{1/0}^{x} - \ln i_{1/0}^{y}}-1) =$$

$$= -\frac{7816,9\cdot 10^9\cdot \ln 1,370565141}{8328,9\cdot 10^9\cdot(\ln 1,694610395 - \ln 1,370565141)}(e^{\ln 1,694610395 - \ln 1,370565141}-1) =$$

$$= -0,329583053$$

As effect, because the model separately described by the road of each factor *X*, respectively *Y*, follow an exponential trend, the influence in absolute sizes of the change concerning the nominal G.D.P., respectively of the variation of actual G.D.P., over the dynamic of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, it was a growth of him with 0,5514808, respectively a subtraction with - 0,329583053.

In continuation, we whish to verify the null hypothesis H_0^{DF**}, through which we supose that, there is not never a significant difference between the multiplication, respectively the sum, of the contributions concerning the separated influences in relative sizes, respectively absolute sizes, of nominal G.D.P. and actual G.D.P., materialized over the variation of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, and determined in the situation in which „factors path" model represents an exponential trend, on a part, and the multiplication, respectively the sum, of the separated contributions in relative sizes, respectively absolute sizes, of the same factors of influence of the dynamic concerning the G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, and calculated in the conditions in which the road cans to express a liner model of trend, on the other part.

Thus, if we hold account by Şaifulin and Seremet opinions [7], through which the model of the road for *X* and *Y* factors from to P_0 towards P_1 evolves after a segment of straight line, then we have the next shapes of arithmetical and geometrical decompositions through „Factors Path Method" concerning the dynamic of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, under the influence of the variation in the same period of time of nominal G.D.P., respectively actual G.D.P.

- the case of the geometrical decomposition:
- the influence in relative sizes of the change of nominal G.D.P. over the variation of G.D.P. Prices Deflator for U.S.A., in 2006 face of 1996, will be:

$$I_{1/0}^{\varphi(x/y)} = e^{\int\limits_{(P_0P_1)} \frac{dx}{x}}$$

- the influence in relative sizes of the variation concerning the actual G.D.P. over the dynamic of G.D.P. Ptices Deflator for U.S.A. in 2006 face of 1996, will be:

$$I_{1/0}^{\varphi(y/x)} = e^{-\int\limits_{(P_0P_1)} \frac{dy}{y}}$$

In the hypothesisi of the liniarity concerning the variation in time of *X* factor, we have:

$$x_{t_i} = A + B \cdot t_i$$

If we put the conditions:

$$x(0) = A \text{ şi } x(1) = A + B = x(0) + B \Rightarrow B = x(1) - x(0) = \Delta_{1/0}^{x}$$

we obtaine:

$$x = x(0) + \Delta_{1/0}^{x} \cdot t \Rightarrow dx = \Delta_{1/0}^{x} \cdot dt$$

If we demonstrate in analogous mode, in the hypothesis of the liner variation for *Y* factor, we have:

$$y = y(0) + \Delta_{1/0}^{y} \cdot t \Rightarrow dy = \Delta_{1/0}^{y} \cdot dt$$

Consequently, we must to take in consideration the fact that:

$$\int\limits_{(P_0P_1)} \frac{dx}{x} = \int\limits_{x(0)}^{x(1)} \frac{dx}{x(0) + \Delta_{1/0}^{x} \cdot t} = \int\limits_{0}^{1} \frac{\Delta_{1/0}^{x}}{x(0) + \Delta_{1/0}^{x} \cdot t} dt = \ln[x(0) + \Delta_{1/0}^{x}] - \ln[x(0)] = \ln[x(1)] - \ln[x(0)] =$$

$$= \ln \frac{x(1)}{x(0)} = \ln i_{1/0}^{x} = \ln \frac{13.246{,}6 \cdot 10^9}{7.816{,}9 \cdot 10^9} = \ln 1{,}694610395 = 0{,}527452859$$

Thus,

$$I_{1/0}^{\varphi(x/y)} = e^{\int\limits_{(P_0P_1)} \frac{dx}{x}} = e^{0{,}527452859} = 1{,}694610395 \cong 1{,}6946 \quad \text{sau} \quad 169{,}46\ \%$$

In analogous mode, we have:

$$\int_{(P_0P_1)} \frac{dy}{y} = \ln i^y_{1/0} = \ln\frac{y(1)}{y(0)} = \ln\frac{11.415{,}3\cdot 10^9}{8.328{,}9\cdot 10^9} = \ln 1{,}370565141 = 0{,}315223166$$

Therefore,

$$I^{\varphi(y/x)}_{1/0} = e^{-\int_{(P_0P_1)}\frac{dy}{y}} = e^{-0{,}315223166} = 0{,}729626028 \cong 0{,}7296 \text{ sau } 72{,}96\ \%$$

Hence, in the situation in which the road described on each factor, *X*, respectively *Y*, represented a model of liner trend, the value of G.D.P. Prices Deflator for U.S.A. growed with 69,46% in 2006 face of 1996, under the influence of the change of the nominal G.D.P., while as follow of the influence concerning the actual G.D.P., the value of G.D.P. Prices Deflator subtracted with – 27,04 % in the same period of time.

- the case of the arithmetical decomposition

- the influence in absolute sizes concerning the variation of the nominal G.D.P. over the change of the value for U.S.A. G.D.P. Prices Deflator, in 2006 face of 1996, it's:

$$\Delta^{\varphi(x/y)}_{1/0} = \int_{(P_0P_1)} \varphi'_x dx = \int_{(P_0P_1)} \frac{1}{y} dx$$

- the influence in absolute sizes of the change concerning the actual G.D.P. over the dynamic of the level for U.S.A. G.D.P. Prices Deflator, in 2006 face of 1996, it's:

$$\Delta^{\varphi(y/x)}_{1/0} = \int_{(P_0P_1)} \varphi'_y dy = -\int_{(P_0P_1)} \frac{x}{y^2} dy$$

If we formulate the hypothesis through which the road of *X*, respectively *Y*, factors, it'a a liner trend, we have:

$$\Delta^{\varphi(x/y)}_{1/0} = \int_{(P_0P_1)} \varphi'_x dx = \int_{(P_0P_1)} \frac{1}{y} dx = \int_{x(0)}^{x(1)} \frac{1}{y(0)+\Delta^y_{1/0}\cdot t} dx = \int_0^1 \frac{\Delta^x_{1/0}}{y(0)+\Delta^y_{1/0}\cdot t} dt = \frac{\Delta^x_{1/0}}{\Delta^y_{1/0}}\cdot[\ln(y(0)+\Delta^y_{1/0}) - \ln(y(0))] =$$

$$= \frac{\Delta^x_{1/0}}{\Delta^y_{1/0}}\cdot[\ln(y(1)) - \ln(y(0))] = \frac{\Delta^x_{1/0}}{\Delta^y_{1/0}}\cdot\ln\left[\frac{y(1)}{y(0)}\right] = \frac{\Delta^x_{1/0}}{\Delta^y_{1/0}}\cdot\ln i^y_{1/0} =$$

$$= \frac{13.246{,}6\cdot 10^9 - 7.816{,}9\cdot 10^9}{11.415{,}3\cdot 10^9 - 8.328{,}9\cdot 10^9}\cdot\ln\frac{11.415{,}3\cdot 10^9}{8.328{,}9\cdot 10^9} = 0{,}554551329$$

Also, we obtaine:

$$\Delta^{\varphi(y/x)}_{1/0} = \int_{(P_0P_1)} \varphi'_y dy = -\int_{(P_0P_1)} \frac{x}{y^2} dy = -\int_{y(0)}^{y(1)} \frac{x(0)+\Delta^x_{1/0}\cdot t}{[y(0)+\Delta^y_{1/0}\cdot t]^2} dy = -\int_0^1 \frac{x(0)+\Delta^x_{1/0}\cdot t}{[y(0)+\Delta^y_{1/0}\cdot t]^2}\cdot\Delta^y_{1/0} dt =$$

$$= -x(0)\cdot\Delta^y_{1/0}\int_0^1 \frac{dt}{[y(0)+\Delta^y_{1/0}\cdot t]^2} - \Delta^x_{1/0}\cdot\Delta^y_{1/0}\int_0^1 \frac{t}{[y(0)+\Delta^y_{1/0}\cdot t]^2} dt =$$

$$= x(0)\cdot\int_0^1 \left(\frac{1}{y(0)+\Delta^y_{1/0}\cdot t}\right)' dt + \Delta^x_{1/0}\cdot\int_0^1 \left(\frac{1}{y(0)+\Delta^y_{1/0}\cdot t}\right)'\cdot t dt =$$

$$= x(0)\cdot\left[\frac{1}{y(0)+\Delta^y_{1/0}} - \frac{1}{y(0)}\right] + \frac{\Delta^x_{1/0}}{y(0)+\Delta^y_{1/0}} - \Delta^x_{1/0}\cdot\int_0^1 \frac{dt}{y(0)+\Delta^y_{1/0}\cdot t} =$$

$$= x(0)\cdot\left[\frac{1}{y(1)} - \frac{1}{y(0)}\right] + \frac{\Delta^x_{1/0}}{y(1)} - \frac{\Delta^x_{1/0}}{\Delta^y_{1/0}}\cdot\int_0^1 \left[\ln(y(0)+\Delta^y_{1/0}\cdot t\right]' dt =$$

$$= -\frac{x(0)\cdot\Delta^{y}_{1/0}}{y(0)\cdot y(1)} + \frac{\Delta^{x}_{1/0}}{y(1)} - \frac{\Delta^{x}_{1/0}}{\Delta^{y}_{1/0}}\cdot\left[\ln[y(0)+\Delta^{y}_{1/0}] - \ln[y(0)]\right] = -\frac{x(0)\cdot\Delta^{y}_{1/0}}{y(0)\cdot y(1)} + \frac{\Delta^{x}_{1/0}}{y(1)} - \frac{\Delta^{x}_{1/0}}{\Delta^{y}_{1/0}}\cdot \ln i^{y}_{1/0} =$$

$$= -\frac{7.816,9\cdot 10^9\cdot(11.415,3\cdot 10^9 - 8.328,9\cdot 10^9)}{8.328,9\cdot 10^9\cdot 11.415,3\cdot 10^9} + \frac{13.246,6\cdot 10^9 - 7.816,9\cdot 10^9}{11.415,3\cdot 10^9} -$$

$$-\frac{13.246,6\cdot 10^9 - 7.816,9\cdot 10^9}{11.415,3\cdot 10^9 - 8.328,9\cdot 10^9}\cdot \ln\frac{11.415,3\cdot 10^9}{8.328,9\cdot 10^9} = -0,332653582$$

Consequently, in the conditions in which the factors X and Y separately vary in time according to a liner model, the effect of the influence in absolute sizes of the change concerning the nominal G.D.P., respectively of the variation for the actual G.D.P., over the dynamic of the value of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, reflects a growth of this with 0,554551329, respectively a subtract with - 0,332653582. If we achieve a comparative analysis between the results obtained concerning the levels of the influences in relative sizes and in absolute sizes of the factors X and Y, namely of the nominal G.D.P. and of the actual G.D.P., over the dynamic of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, when the separetely model of the road for each factor it's an exponential trend, and in situation in which the roads of the factors X and Y from to P_0 towards P_1 describe the liner trend according to the opinions of Şaifulin and Seremet, we obtaine the next „picture of board" concerning the dates presented in the table number 2:

Table no. 2 The comparative analysys of the results when the road of the factors *X* and *Y* it's an exponential trend and in the case when the models concerning the roads of the factors *X* and *Y* follow liner trends

The size of the influence / **The model of trend**	**The influence in relative sizes**		**The influence in absolute sizes**		**The multiplication between the influence in relative sizes of *X* factor and the influence in relative sizes of *Y* factor**	**The sume between the influence in relative sizes of *X* factor and the influence in relative sizes of *Y* factor**
	***X* factor** (nominal G.D.P.)	***Y* factor** (actual G.D.P.)	***X* factor** (nominal G.D.P.)	***Y* factor** (actual G.D.P.)		
Exponential	1,694610395	0,729626028	0,551480800	- 0,329583053	1,236431852	0,221897747
Liner	1,694610395	0,729626028	0,554551329	- 0,332653582	1,236431852	0,221897747

But,

- the index of the variation concerning G.D.P. Prices Deflator for U.S.A. as effect of the influence of the variations for the both factors (nominal G.D.P. and actual G.D.P.), in 2006 face of 1996, will be:

$$I^{\varphi(x\cup y)}_{1/0} = \frac{I^{p}_{dfl,1}}{I^{p}_{dfl,0}} = \frac{1,160425044}{0,938527297} = 1,236431852$$

where: $I^{\varphi(x\cup y)}_{1/0}$ = the index of the variation concerning G.D.P. Prices Deflator for U.S.A. under the influences of the variations for the both factors (nominal G.D.P and actual G.D.P.);

$I^{p}_{dfl,0}$ = the value of G.D.P. Prices Deflator for U.S.A. in 1996;

$I^{p}_{dfl,1}$ = the size of G.D.P. Prices Deflator for U.S.A. in 2006.

- the absolute change of G.D.P. prices Deflator for U.S.A. as effect of the variations for the both factors (nominal G.D.P. and actual G.D.P.), in 2006 face of 1996, will be:

$$\Delta^{\varphi(x\cup y)}_{1/0} = I^{p}_{dfl,1} - I^{p}_{dfl,0} = 1,160425044 - 0,938527297 = 0,221897747$$

We observe that, both in the situation in which the road of each fator reflects a model of exponential trend, and in the case in which the models described by nominal G.D.P. and actual G.D.P. expresse liner roads of the factors, the multiplication of the factorial indexes it's equal with the general index, namely with the index of the variation concerning the G.D.P. Prices Deflator for U.S.A. under the influence of the both factors (nominal G.D.P. and actual G.D.P.), in 2006 face of 1996:

$$I^{\varphi(x\cup y)}_{1/0} = I^{\varphi(x/y)}_{1/0}\cdot I^{\varphi(y/x)}_{1/0}$$

or $$1{,}236431852 = 1{,}694610395 \cdot 0{,}729626028$$

On the other share, in the cases in which the roads followed by the nominal G.D.P. and actual G.D.P., reflect in a separated mode by an exponetial trend, the sume of the separated factorial influences in absolute sizes, manifested over the dynamic of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, it's equal with the variation in absolute sizes of the level concerning the G.D.P. Prices Deflator for U.S.A. under the influence of the both factors, in the same period of time:

$$\Delta_{1/0}^{\varphi(x \cup y)} = \Delta_{1/0}^{\varphi(x/y)} + \Delta_{1/0}^{\varphi(y/x)}$$

or $$0{,}221897747 = 0{,}5514808 + (-0{,}329583053)$$

Also, we observe that and in the conditions in which the models of the roads followed by the nominal G.D.P and actual G.D.P., represented the liner trends, the sume of the separated factorial influences in absolute sizes, achived over the dynamic of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, it's equal with the total variation of the level concerning the G.D.P. prices Deflator for U.S.A. in absolute sizes under the influence of the both factors, in the respective period of time taked in observation:

$$\Delta_{1/0}^{\varphi(x \cup y)} = \Delta_{1/0}^{\varphi(x/y)} + \Delta_{1/0}^{\varphi(y/x)}$$

or $$0{,}221897747 = 0{,}554551329 + (-0{,}332653582)$$

So, we can to specify that, both in the situation in which the two factors of influence of the dynamic concerning the G.D.P. Prices Deflator, follow in a separated mode a road which reflect a model of exponential trend, and in the situation in which each factor describes a road of shape of a model of liner trend, it se manifests the equalities between the multiplication, respectively the sume, of the separated influences in relative sizes, respectively absolute sizes of the nominal G.D.P. and the actual G.D.P. achieved over the dynamic of G.D.P. prices Deflator for U.S.A. in 2006 face of 1996, on a share, and the level of the total variation, in relative sizes, respectively absolute sizes, in the same period of time, of a G.D.P. Prices Deflator for U.S.A. as effect of the variations for the both factors: the nominal G.D.P. and actual G.D.P., on a other share, which it demonstrated that it verifies the null hypothesis $H_0^{DF^{**}}$.

3. Conclusions

In conclusion, the opinions of Seremet and Şaifulin are valid in the case of this research, only from the point of view of the observed equalities in the situation in which each factor varies according to a liner function, namely, the multiplication, respectively the sume, of the values of the separated influences in relative sizes, respectively absolute sizes, of nominal G.D.P. and actual G.D.P., over the variation of G.D.P. Prices Deflator for U.S.A. in 2006 face of 1996, there are equal with the level of the total relative dynamic, respectively the total absolute dynamic, in the same period of time, of G.D.P. Prices Deflator for U.S.A. as effect of the changes for the both factors: nominal G.D.P. and actual G.D.P. Yet, in this research, the road of each factor represented by the nominal G.D.P., respectively the actual G.D.P., between P_0 and P_1, it's a model of exponential trend, because the method of the coefficient of variation reflects that the road followed in a separated mode by each factor X and Y, respectively the nominal G.D.P. and the actual G.D.P, between 1996 and 2006, it's of the shape of the exponential trend, witch it grounds Ioan Florea opinion.

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